

THE VISCOPLASTICITY THEORY BASED ON OVERSTRESS
APPLIED TO THE MODELING OF A NICKEL BASE SUPERALLOY AT 815°C

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Short-term strain rate change, creep and relaxation tests were performed in an MTS computer-controlled servohydraulic testing machine. Aging and recovery were found to be insignificant for test times not exceeding thirty hours. The material functions and constants of the theory were identified from results of strain rate change tests. Numerical integration of the theory for relaxation and creep tests showed good predictive capabilities of the viscoplasticity theory based on overstress.

Introduction

Advanced materials are being developed for use in high temperature gas turbine applications. To fully utilize the capability of these new materials their deformation properties and their creep and fatigue fracture characteristics must be determined by suitable experiments. The experimental findings must be analyzed, idealized and translated into constitutive equations for use in stress analysis and life prediction. Only when these ingredients together with appropriate computational tools are available can durability analysis be performed in the design stage long before a component is being built. This paper contributes to the design methodology and reports on an experimental investigation of the deformation behavior of a nickel-base superalloy at 815°C and on its mathematical modeling using the viscoplasticity theory based on overstress.

Testing Method and Test Materials

All tests were performed in an MTS axial-torsion servohydraulic mechanical testing machine with an MTS 463 Data/Control processor for computer control. An MTS clamshell furnace was used to heat the specimens. Strain measurement on the gage section was done with an MTS high-temperature uniaxial extensometer. Strain and load controlled tests were performed at 815°C. Separate tests checked on the extensometer calibration (at room temperature) and on the uniformity of the temperature along the gage length. For the duration of the tests the temperature of the gage section stayed within 2°C of the nominal temperature.

The nickel-base superalloy test material was donated by AVCO Lycoming and was delivered in the form of 19 mm coupons. The machined specimens had a gage section diameter of about 6.5 mm. They were tested in the as-received condition at 815°C.

Experimental Results

Effects of strain rate

Figure 1 shows the results of three tests: two were subjected to continuous straining at 10^{-4} and 10^{-6} s $^{-1}$, respectively; the other to a sequential decrease in strain rates. The significant influence of strain rate is evident. These and other strain rate change tests revealed no discernible strain rate history effect. The strain rate change tests in Fig. 1 and others indicate that there is a unique stress level associated with each strain rate which will be reached after a transient period

irrespective of the prior history. The stress levels were determined from the tests and are listed in Table 1 together with the stress level differences relative to the stress corresponding to 10^{-4} s^{-1} .

The tests in Fig. 1 show good reproducibility which was also found with others. An exception is the stress level at 10^{-6} s^{-1} for specimens #12 and #15. At 1.1 percent strain, specimen 15 suffered a temperature variation of about 4°C which may have caused the drop-off at point A. The filled triangle indicates the stress level at 10^{-7} s^{-1} found in other experiments. Premature cracking is probably the cause for the decrease in stress level of specimen #7 before unloading started.

Aging and recovery

To ascertain whether these phenomena have a significant influence on the deformation behavior two specimens were subjected to a 3 and 33 hours hold period, respectively, at zero load after loading to 1 percent strain and subsequent unloading had been accomplished. It is seen from Fig. 2 that the stress-strain curves before and after the rest period are within the normal scatter of the test results. This can be easily ascertained by comparing the responses of the two specimens and the results of Figs. 1 and 2.

The tests show therefore that recovery and aging are insignificant for this material for times less than 33 hours at 815°C . This result was very surprising to the authors.

Modeling. The Viscoplasticity Theory Based on Overstress (VBO)

The model and its material functions

In the present version of VBO recovery and aging are not included. In the uniaxial state of stress the theory consists of two coupled, nonlinear, ordinary differential equations which contain two positive, decreasing material functions; the viscosity function k controls the rate dependence and the shape function ψ influences the shape of the knee of the stress-strain curve. In addition, the elastic modulus E , the tangent modulus E_t and the asymptotic value A of the equilibrium stress g enter into the theory. The equilibrium stress is attained in the limit as rates approach zero. The difference $\sigma - g = x$ between the stress σ and g is called the overstress and k and ψ are only functions of x . The equations are

$$d\epsilon/dt = d\epsilon^{el}/dt + d\epsilon^{in}/dt = d\sigma/dt/E + (\sigma - g)/(Ek[x]) \quad (1)$$

$$dg/dt = \psi[x]d\epsilon/dt - (g - E_t\epsilon)(\psi[x] - E_t)|d\epsilon^{in}/dt|/A \quad (2)$$

where square brackets following a symbol denote "function of." Under constant strain rate loading, the system of equations permits asymptotic solutions given by

$$\{d\sigma/d\epsilon\} = \{dg/d\epsilon\} = E_t \quad (3)$$

$$\{x\} = (E - E_t) k[\{x\}]d\epsilon/dt \quad (4)$$

$$\{g - E_t\epsilon\} = A d\epsilon/dt / |d\epsilon^{in}/dt| \quad (5)$$

where braces denote asymptotic values. Details of the theory are found in [1,2].

The asymptotic solutions are algebraic equations and are used in identifying the material constants and functions. The procedure exploits (4) to compute the stress

level differences at different strain rates so that the data of Table 1 can be used. Details can be found in [3]. The g-curve is obtained by extrapolation and a nonlinear least square analysis is performed to determine the constants of the shape function. The material functions and constants obtained by this method are given in Table 2.

Predictions of the theory

With the material constants determined from strain rate change tests, the theory was applied to predict the outcome of relaxation and creep tests. Figure 3 shows the results of a strain rate change test followed by relaxation tests of 1024 s duration. The triangles represent experimental results; the continuous line, the predictions of the theory as obtained by integrating (1) and (2) numerically using the IMSL routine DGEAR. The prediction for incremental creep tests is depicted in Fig. 4. The creep periods last 700 s except at the highest stress level where creep was terminated after 300 s. The experimental results are again given as triangles.

The discrepancy between predictions and experiment in Fig. 3 is mainly due to an overprediction of relaxation in the first relaxation period aa'. If allowance is made for this difference and the theoretical curves are translated upwards so that they coincide at the end of the first relaxation period, the subsequent predictions are very reasonable. A similar observation holds for the stress vs time relaxation curves.

It is seen that both the theory and the experiment do not show creep at stress levels a and b in Fig. 4. At the next two stress levels the theory underpredicts the creep strain but overpredicts it at stress levels e and f.

The fitting of the material functions was only done once on the basis of the strain rate change test results. Both the creep and the relaxation experiments suggest that the theory overpredicts at high inelastic strain rates. Since underprediction is observed at the small stress levels in Fig. 4, it is reasonable to assume that an optimization of the material functions is possible so as to improve the predictions. This optimization and other tests are planned for the future.

Acknowledgment

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References

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TABLE 1

Averaged Flow Stress Levels and Stress Level Differences
for C101 at 815°C

Strain Rate s^{-1}	Stress Level at 1.2% MPa	Stress Level Difference MPa
10^{-3}	966	132
10^{-4}	834	0
10^{-5}	721	-113
10^{-6}	638.4	-195.6
10^{-7}	554.4	-279.6

TABLE 2

THE DETERMINED MATERIAL CONSTANTS AND FUNCTIONS

<u>Material Constants</u>	<u>Shape Modulus</u>
$E = 156620 \text{ MPa}$	$\psi[x] = c_1 + (c_2 - c_1)\exp(-c_3 x)$
$E_t = 267 \text{ MPa}$	c_1, c_2 and x in units of MPa;
$A = 421.7 \text{ MPa}$	$c_1 = 62500, c_2 = 150000$
<u>Viscosity Function</u>	c_3 in units of MPa^{-1} ;
$k[x] = k_1 \left(1 + \frac{ x }{k_2}\right)^{-k_3}$	$c_3 = 0.0387$
$k_1 = 3150000 \text{ s}$	
$k_2 = 186 \text{ MPa}$	
$k_3 = 9.96$	

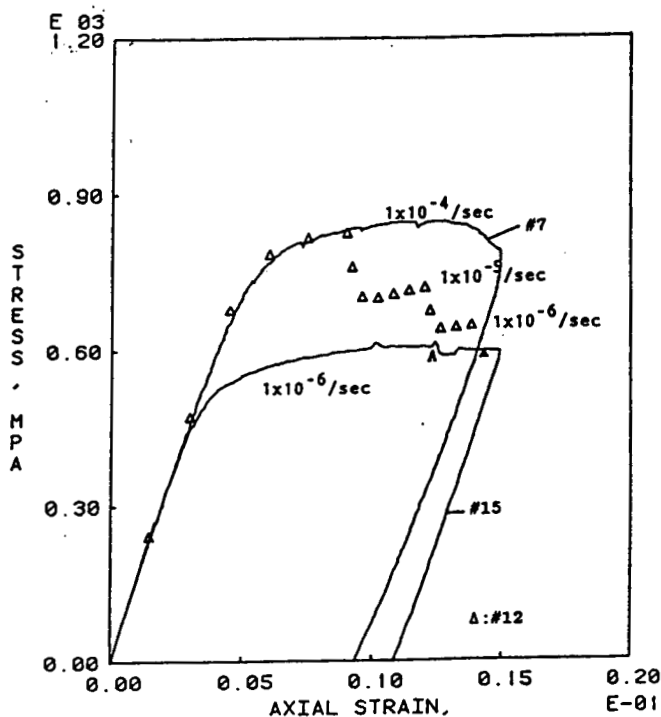


Figure 1. Stress-strain curves of constant strain-rate test (solid lines) and strain-rate change tests (triangles). Filled triangle denotes the approximate stress level at 1×10^{-7} per second.

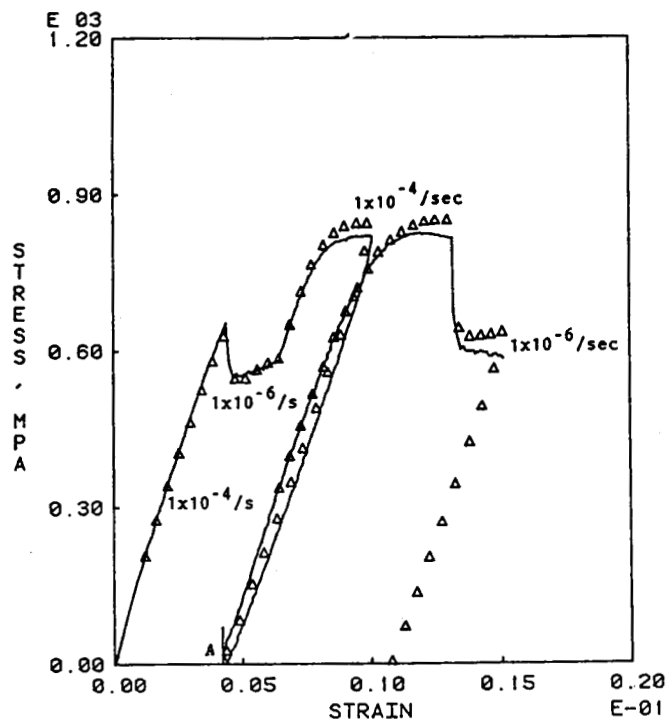


Figure 2. Stress-strain curves of specimens #8(-) and #10(Δ). Hold periods of 3 and 33 hrs at A.

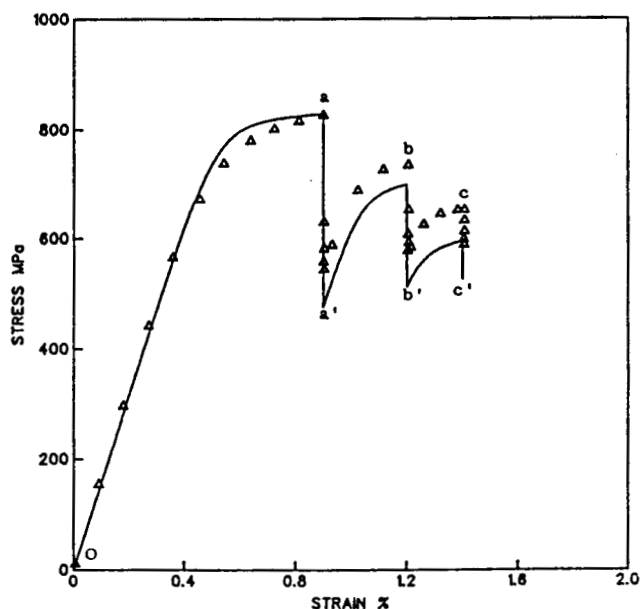


Figure 3. Prediction of strain-rate change and relaxation tests. Triangles represent experimental results. Strain-rate history: oa 10^{-7} , aa' 0, $a'b$ 10^{-5} , bb' 0, $b'c$ 10^{-6} , cc' 0. All values are given in units of s^{-1} . Relaxation time = 1024 s.

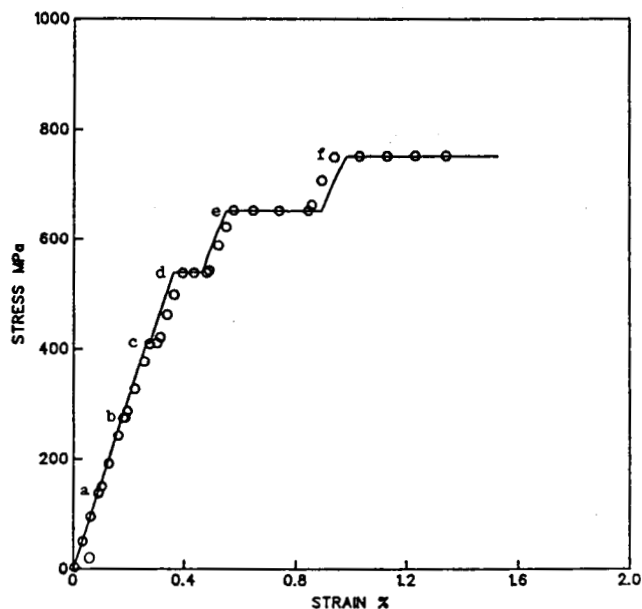


Figure 4. Incremental creep tests at various stress levels. Between creep periods the stress rate is 2.8 MPa/s. Almost no creep is observed at the two stress levels below 300 MPa. Creep periods are 700 s except at level 4. Circles represent experimental points.